FLUID MECHANICS IN THE SUBTECTORIAL SPACE

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In the subtectorial space, momentum is transported by the fluid to the stereocilia of the inner hair cell (IHC), resulting in bending of the hair bundle. The fluid must pass through the v-shaped arrangement of the outer hair cell (OHC) stereocilia and is "squeezed" between the tectorial membrane (TM) and the reticular lamina, especially due to the Hensen's stripe. Here, we analyze the flow field by means of numerical discretization. Since the geometry is complex, the finite-element-method is employed. An implementation is developed which allows computation in the frequency domain, based on a pressure stabilised velocity formulation. Matrix coupling with the structure is accomplished and a single solution step of the whole system matrix is sufficient to retrieve the displacement field of the fluid and structural parts. The proposed method allows a solution of the fluid-structure interaction in the subtectorial space within a reasonable time. For a two-dimensional mesh with 120,000 degrees of freedom of a complete cross-section of the organ of Corti, including stereocilia, the solution time is less than 20 seconds on a convential PC.

1. Introduction

Here, a novel technique is proposed to model fluid-structure-interaction problems which are common in the field of inner-ear mechanics. It is assumed that the non-linear term in the Navier-Stokes equation can be neglected. In other words, the Reynolds number is smaller than unity, so that a computation of acoustic streaming is not necessary [1]. Although the dimensions are small, we assume that a continuum approach is still valid for the geometrical dimensions of the order of one micrometer. The no-slip condition is used at the boundaries, since the Debye-length is about one nanometer [2] and, therefore, much smaller than the geometry.

Furthermore, we perform a discretisation of the linear viscous fluid with near-incompressibility. In this respect, the only assumptions are that the amplitudes are small and the wavelength long. Estimations about the flow field are not necessary, as they are in lumped models for the fluid in a gap [3, 4]. The linear approach allows a much faster solution than with staggered techniques, such as those used by Givelberg and Bunn [5].

After presenting the method, it is applied to a two-dimensional cross-section model of the guinea-pig at a characteristic place-frequency of 0.8 kHz. Coiling effects of the cochlea are neglected [6, 7], because we are examining vibration responses from the boundaries of the subtectorial space, directly in response to somatic electromechanical forces from the

OHCs.

2. Methods

2.1. *Fluid-Structure-Interaction*

For small amplitudes the non-linear term of the Navier-Stokes equations can be neglected. If the problem is isothermal and variations of pressure, p, and density, ϱ, are small compared to their mean value, a nearly incompressible fluid can be assumed [8]. The mass-conservation equation

$$\frac{1}{K}\frac{\partial p}{\partial t} + \nabla \cdot \vec{v} = 0 \tag{1}$$

relates the time derivative of the pressure divided by the compressibility K to the divergence of the velocity vector $\vec{v}$. The linear part of the momentum equation without volume forces

$$\varrho \frac{\partial \vec{v}}{\partial t} = -\nabla\, p + \nabla \cdot \tau \tag{2}$$

relates the acceleration to the gradient of pressure and stresses. A Stokes-fluid is described by

$$\tau = \mu \left(\nabla\, \vec{v} + (\nabla\, \vec{v})^{\mathrm{T}} - \frac{2}{3}\, \nabla \cdot \vec{v} \right) \tag{3}$$

where μ is the dynamic viscosity. The unknown variables are the pressure, p, and the velocities, $\vec{v}$.

For an efficient coupling with the solid structure, the velocity is substituted by the time derivative of the displacement $\vec{u}$

$$\vec{v} = \frac{\partial \vec{u}}{\partial t}\ . \tag{4}$$

Applying a finite element discretisation to Eqns. (1,2,4) yields a set of linear equations for the unknown degrees of freedom:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{p}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & -\mathbf{V} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix} \tag{5}$$

for given forces $\mathbf{f}$. Here $\dot{(\,)}$ denotes the first time derivative $\partial(\,)/\partial t$ and $\ddot{(\,)}$ the second. The matrix $\mathbf{A}$ describes the shearing and is proportional to the dynamic viscosity μ. $\mathbf{V}$ ensures the nearly incompressibile material and depends on the compressibility K. $\mathbf{C}$ is the coupling between pressure and velocity and $\mathbf{M}$ is the inertia and depends on the fluid density ϱ. Details are given by Zienkiewicz *et al.* [9]. After elimination of the pressure variable, the system

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{A}\,\dot{\mathbf{u}} + \mathbf{C}\,\mathbf{V}^{-1}\,\mathbf{C}^{\mathrm{T}}\,\mathbf{u} = \mathbf{f} \tag{6}$$

describes a fluid simply in terms of displacement. This representation allows a direct implementation in a finite element solver for structure problems and leads to a monolithic fluid-structure approach. A time-harmonic Ansatz is possible for the linear system. Thus, for excitation with a single frequency, the system given by Eqn. (6) must be solved only once. Here, this fluid element is based on quadratic shape functions for displacement and geometry. Linear shape functions are used for the pressure to ensure stability. The element is implemented as an user-defined element in Ansys [10].

The correct implementation is validated by the analytical solution for the oscillating motion of an infinite cylinder normal to its axis [11]. This problem involves inertia and viscous forces on a curved geometry. There is good agreement in the entire flow field, provided the unsteady boundary layer is resolved. The comparison is plotted in Fig. 1 for the amplitude and phase of the circumferential and radial velocity component.

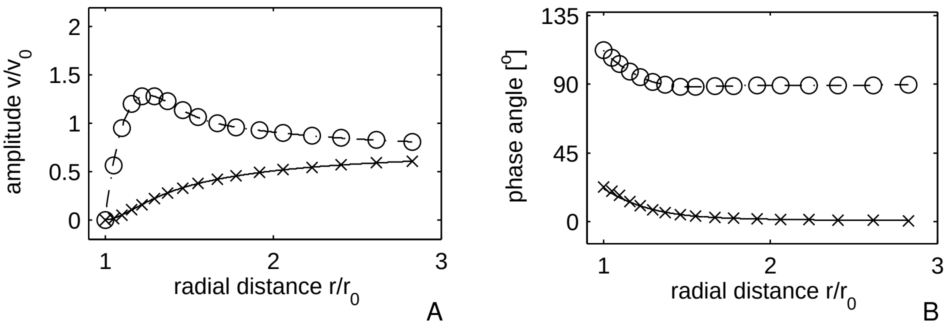

Fig. 1. Comparison of the velocities around an oscillating cylinder normal to its axis with a Womersley number of 10. Velocity profile in radial direction, 45° off the main axis. Analytical solution for the peripheral (— · —) and radial (——) direction and numerical results at the nodes ∘ and ×, respectively. (A) Amplitude. (B) Phase angle.

2.2. *Geometry*

The numerical model is built on geometrical data from the guinea-pig cochlea at a characteristic place-frequency of 0.8 kHz. Data derive from live-stained (FM 1-43, Invitrogen) and TM-stained (Lectin SBA Conjugates, Invitrogen) preparations and stacks of images obtained from laser scanning microscope Leica SP5 (Fig. 2). The geometry is manually constructed and represented by splines. Each cell type and the fluid spaces are grouped into closed areas. A set of mechanical properties is assigned and then meshed with finite elements (quadratic shape functions).

2.3. *Properties*

The elasticity of the material is based on data used by Steele and Puria [4] and adjusted to match experimental data from different sources. The measurements of Cooper [12] give an indication for the stiffness distribution of the basilar membrane. The OHCs are matched to data from Frank *et al.* [13] and recent impedance measurements [14]. The stiffness of the hair bundle is based on values from Strelioff and Flock [15]. The impedance along the reticular lamina is similar to values measured by Scherer and Gummer [16]. The fluid

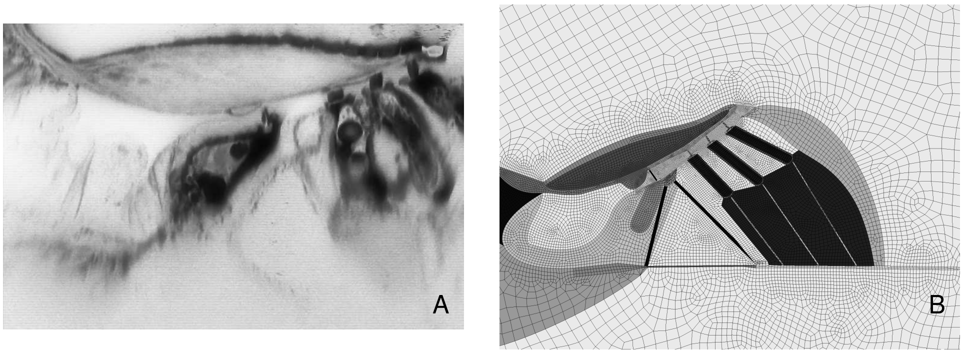

Fig. 2. Cross section of the guinea-pig organ of Corti at a characteristic place-frequency of 0.8 kHz. (A) Live staining with FM 1-43 (Invitrogen). Image is extracted from a stack. (B) Geometry and mesh for the computation. Here, the different grey levels just indicate different areas and do not correspond to physical properties.

damping was not sufficient to match these data and additional material damping in the Hensen and sulcus cells was necessary. According to the experiments of Ghaffari *et al.* [17], the TM is highly viscous. The vibrational measurements at the boundary of the subtectorial space by Nowotny and Gummer [18] serve here as reference for the motion around the IHC stereocilia. The motion of the cross-section is compared with measured displacements [19, 20].

The dynamic viscosity of the fluid is 1 Pa ms[a], as measured at room temperature for Hanks solution [1]. The density for all materials is set to 0.001 ng/μm^3. The OHCs are modelled with a piezoelectric wall and are filled with incompressible fluid. The elasticity of the basilar membrane is set to yield a resonance frequency of 0.8 kHz for a constant pressure load radially across the membrane. Comparisons are made with experimental vibration data from the TM and reticular lamina in an in-vitro preparation in which the TM was intact, with the longest OHC stereocilia embedded in the TM [18].

3. Results

The numerical method was applied to solve a fluid-structure interaction in the organ of Corti. The two-dimensional mesh has about 60,000 nodes. Ansys [10] was used as finite element software. The approach is based on non-moving meshes and has no limitations due to small amplitudes.

To model the motion of the organ of Corti in response to somatic electromechanical force from the OHCs, a sinusoidal voltage is applied across the OHC wall. Due to the assigned piezoelectric property of the cell wall, the applied voltage sinusoidally modulates the wall thickness. In turn, the axial length of the cell is modulated because the intracellular fluid is incompressible. This excitation causes a pattern of motion, as shown in Fig. 3.

The inner and outer pillar cells are not connected to each other in the longitudinal

[a]The following basis units are used: mass in nanogram (ng), length in micrometer (μm), time in millisecond (ms). Thus, the pressure is in Pa.

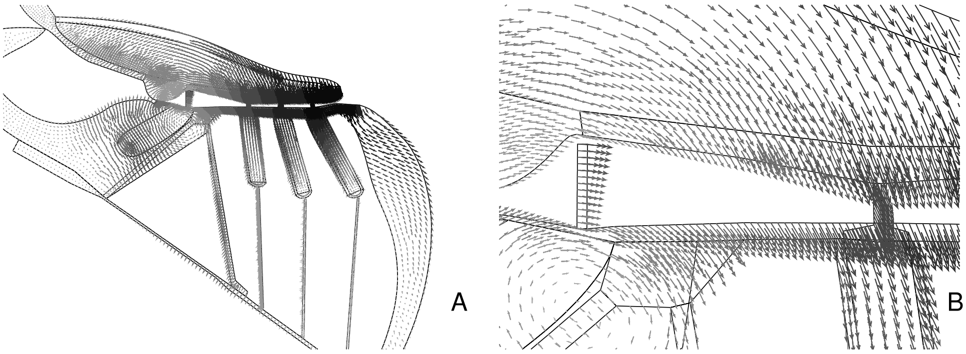

Fig. 3. Displacement of the organ of Corti at a characteristic place-frequency of 0.8 kHz (A) and a close up around the IHC stereocilia (B), at the time instant of maximum OHC contraction.

direction. Thus, there can be fluid flow through the rows of OHCs and pillar cells. The fluid spaces are connected to model this here in two dimensions.

The IHC stereocilia bending amplitude and direction depend on the size of the gap between Hensen's stripe and the reticular lamina. The motion of the subtectorial space at the IHC is characterized by an anti-phasic transversal motion of the reticular lamina and TM, as also found experimentally [18]. For a sufficiently small gap (here ≈ 2μm), motion of the reticular lamina towards the TM at the IHC causes deflection of the IHC stereocilia in the excitatory (depolarizing) direction.

4. Discussion

Fluid flow in the subtectorial space cannot be investigated without considering the motion of the neighbouring structures. The proposed method allows fast computation of strongly coupled fluid-structure interaction problems. For a two-dimensional model of the organ of Corti with around 120,000 degrees of freedom – that is, with two degrees of freedom for each of the 60,000 nodes –, the solution of the problem requires less than 20 seconds for one frequency on a conventional PC (AMD Athlon 64 3500+, 2GB RAM, SuSE Linux 10.3).

Application of the proposed method to a two-dimensional model of the organ of Corti reveals deflection of the IHC stereocilia in the excitatory direction for contraction of the OHC. This motion is caused by a squeezing flow between Hensen's stripe and reticular lamina. This phase relation is found experimentally by Chiaradia *et al.* [21] and the influence of Hensen's stripe has been addressed by Steele and Puria [4].

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References

1. J. Baumgart and R. Grundmann. Fluid motion in the organ of corti. In *19th International Congress on Acoustics*, 2007.
2. J. Baumgart, M. Fleischer, Y. Yarin, and R. Grundmann. Fluid flow around the stereocilia. In *8th International Symposium on Experimental and Computational Aerothermodynamics of Internal Flows*, 2007.
3. P.J. Kolston. Comparing in vitro, in situ, and in vivo experimental data in a three-dimensional model of mammalian cochlear mechanics. *Proc Natl Acad Sci U S A*, 96(7):3676–3681, 1999.
4. C.R. Steele and S. Puria. Force on inner hair cell cilia. *International J. Solids and Structures*, 42(21-22):5887–5904, 2005.
5. E. Givelberg and J. Bunn. A comprehensive three-dimensional model of the cochlea. *Journal of Computational Physics*, 191(2):377–391, 2003.
6. H. Cai, D. Manoussaki, and R. Chadwick. Effects of coiling on the micromechanics of the mammalian cochlea. *Journal of The Royal Society Interface*, 2(4):341–348, 2005.
7. D.Manoussaki, R.S. Chadwick, D.R. Ketten, J. Arruda, E.K. Dimitriadis, and J.T. O'Malley. The influence of cochlear shape on low-frequency hearing. *Proc Natl Acad Sci U S A*, 105(16):6162–6166, 2008.
8. O.C. Zienkiewicz, R.L. Taylor, and J.Z. Zhu. *The Finite Element Method: For Fluid Dynamics*. Elsevier, 2006.
9. O.C. Zienkiewicz, R.L. Taylor, and J.Z. Zhu. *The Finite Element Method: Its Basis & Fundamentals*. Elsevier, 2006.
10. Ansys Inc. *Ansys ver.11*. www.ansys.com, 2007.
11. C.-Y. Wang. On high-frequency oscillatory viscous flows. *Journal of Fluid Mechanics*, 32(1):55–68, 1968.
12. N.P. Cooper. Radial variation in the vibrations of the cochlear partition. In *Recent Developments in Auditory Mechanics*, 1999.
13. G. Frank, W. Hemmert, and A.W. Gummer. Limiting dynamics of high-frequency electromechanical transduction of outer hair cells. *Proc Natl Acad Sci U S A*, 96(8):4420–4425, 1999.
14. T. Eckrich, M. Nowotny, C. Harasztosi, M. Scherer, and A.W. Gummer. Impedance measurements of isolated outer hair cells. In *Thirty-First Annual Midwinterresearchmeeting of the Association for Research in Otolaryngology*, 2008.
15. D. Strelioff and A. Flock. Stiffness os sensory-cell hair bundles in the isolated guinea pig cochlea. *Hearing Research*, 15(1):19–28, 1984.
16. M.P. Scherer and A.W. Gummer. Vibration pattern of the organ of corti up to 50 khz: evidence for resonant electromechanical force. *Proc Natl Acad Sci U S A*, 101(51):17652–17657, 2004.
17. R. Ghaffari, A.J. Aranyosi, and D.M. Freeman. Longitudinally propagating traveling waves of the mammalian tectorial membrane. *Proc Natl Acad Sci U S A*, 104(42):16510–16515, 2007.
18. M. Nowotny and A.W. Gummer. Nanomechanics of the subtectorial space caused by electromechanics of cochlear outer hair cells. *Proc Natl Acad Sci U S A*, 103(7):2120–2125, 2006.
19. K.D. Karavitaki and D.C. Mountain. Imaging electrically evoked micromechanical motion within the organ of corti of the excised gerbil cochlea. *Biophys J*, 92(9):3294–3316, 2007.
20. D.K. Chan and A.J. Hudspeth. Mechanical Responses of the Organ of Corti to Acoustic and Electrical Stimulation In Vitro. *Biophys. J.*, 89(6):4382–4395, 2005.
21. C. Chiaradia, M. Nowotny, and A.W. Gummer. Deflection of ihc stereocilia in response to somatic ohc electromotility. In *Mechanics of Hearing Workshop, Keele*, 2008.